B.Sc Part I (Physics Hons)
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Q (a) What are generalised co-ordinates? What is the ~~a~~ advantge of using them?

(b) Set up Lagrangian function for a simple pendulum and hence obtain its equation of motion.

Ans (a) GENERALISED CO-ORDINATES :-

As we known the position and configuration of a system can be specified by using ~~Ca~~ Cartesian co-ordinates. But use of only cartesian co-ordinates sometimes becomes much complicated as it may not be convenient to express even the motion of a single particle in this system. e.g for a particle moving in a central force field in a plane it will be move convenient to use plane polar cordinates r and θ instead of ~~x, y and r~~ x and y, r and θ are connected to x and y the relation.

$$r = \sqrt{x^2+y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{y}{x} \qquad \text{------------ (1)}$$

Similarly we may use spherical polar co-ordinates (r, θ, φ) when the force is spherically symmetric. Then the transformation relation are as follows :-

$$r = \sqrt{x^2+y^2+z^2} \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{y}{x}\right) \quad \text{--- (2)}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2-y^2}}{z}\right)$$

So, we may use different types of coordinates. But the number of coordinates used must be sufficient in number to define the configuration of the system uniquely. These co-ordinates are called generalised co-ordinates and may be defined as

Any set of independent variable sufficient in number of ~~to define~~ to define unambiguously the system of configuration known as generalised coordeniate. In order to choose a suitable set of generalised co-ordeniate in are given problem, we must be guided by the following three principles

(i) Their values determine the configuration of the system.

(ii) They may be varied arbitrarily and independently without violating the constraints on the system

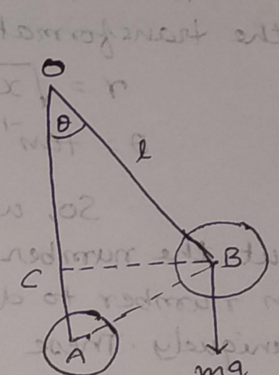
(iii) They must simplify mathematical calculation without affecting the prospects of obtaining an informative and enlightening solution.

The generalised co-ordinates are denote by a symbol q_m

$q_1, q_2 - - - q_m$.

It is the position to express the configuration of a system in cartesian co-ordinates through generalised co-ordinates. Advantage of using generalised co-ordinates -- unlike cartesian co-ordinate, generalised co-ordinates are not divided into these convenient group of co-ordinates which may from vectors. Due to this simplicity generalised coordinates are very helpful in solving the mechanical problems.

(a) SIMPLE PENDULUM :-



Let us suppose that the length of simple pendulum is 'l' and it is suspended from a point O. 'A' denotes the rest position of the pendulum and B is deflected position. The angle θ between the rest position and deflected position is chooses as generalised co-ordenate. The K.E is given by

$$K.E (T) = \frac{1}{2} mv^2 = \frac{1}{2} m(l\dot{\theta})^2 = \frac{1}{2} m l^2 \dot{\theta}^2$$

where m = mass of bob

To calculate the P.E of the system we consider that in coming from position B to A. The bob (of mass m) has fallen freely through a distance vertical CA.

$$P.E\ (V) = mg(OA - OC)$$

$$= mg(l - l\cos\theta) = mgl(1-\cos\theta)$$

So Lagrangian is

$$L = T - V = \frac{1}{2} m l^2 \theta^2 - mgl(1-\cos\theta)$$

$$\therefore \frac{\delta L}{\delta \theta} = m l^2 \theta^2 \text{ and } \frac{\delta L}{\delta \theta} = -mgl\sin\theta$$

Substituting these value in Lagrange's equations

$$\frac{d}{dt}\left\{\frac{\delta L}{\delta \theta}\right\} - \frac{\delta L}{\delta \theta} = 0$$

$$\Rightarrow \frac{d}{dt}(m l^2 \theta^2) + mgl\sin\theta = 0$$

$$\Rightarrow m l^2 \theta^2 + mgl\sin\theta = 0$$

$$\Rightarrow \theta^2 + \frac{g}{l}\sin\theta = 0$$

for small oscillations of the pendulum $\theta \simeq \theta$

$$\theta^2 + \frac{g}{l}\theta = 0$$

This is the required equation of motion of simple pendulum and its time period is

$$T = 2\pi\sqrt{\frac{l}{g}}$$